4-colorability of P6-free graphs

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Chromatic Number

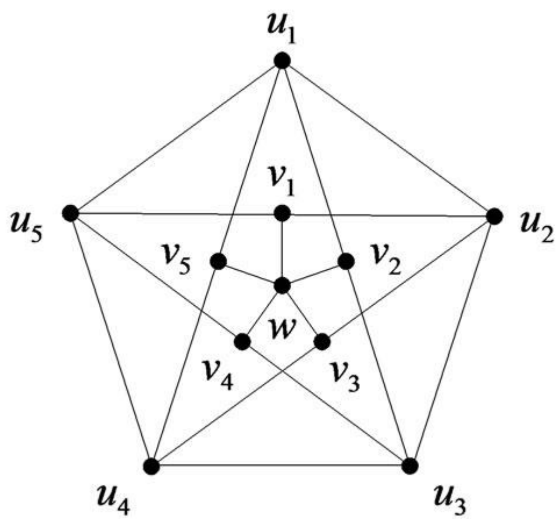
Theorem (Randerath, IS, and Tewes, 2002)

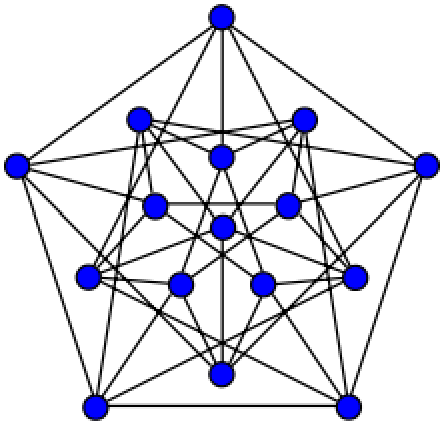
The 3-colorability problem can be solved in polynomial time for P_5-free graphs.

Theorem (Randerath and IS, 2004)

The 3-colorability problem can be solved in polynomial time for P_6-free graphs.

Chromatic Number





Theorem (Chudnovsky et al., 2014)

The 3-colorability problem can be solved in polynomial time for P_7-free graphs.

Chromatic Number

Theorem (Hoang, Kaminski, Lozin, Sawada, and Shu, 2010)

The k-colorability problem can decided in polynomial time for P_5-free graphs.

Theorem (Huang, 2013)

The k-colorability problem is NP-complete for the class of P_t-free graphs for all $k \geq 5, t \geq 6$.

The 4-colorability problem is NP-complete for the class of P_t-free graphs for all $t \geq 7$.

Chromatic Number

Theorem (Kaminski and Lozin, 2007)

For every $k, g \geq 3$, the k-colorability problem for graphs with no cycles of length at most g is NP-complete.

Theorem (Kaminski and Lozin, 2007)

Let H be a graph containing a cycle. For every k, the k-colorability problem for H-free graphs remains NP-complete.

Chromatic Number

Theorem (Kaminski and Lozin, 2007)

For every $k \geq 3$, the k-colorability problem for claw-free graphs is NP-complete.

Theorem (Kaminski and Lozin, 2007)

Let H be a graph containing a claw. For every k, the k-colorability problem for H-free graphs remains NP-complete.

Chromatic Number

Theorem (Kaminski and Lozin, 2007)

Let $k \geq 3$ be an integer, and H a graph. If the k-colorability problem for H-free graphs can be solved in polynomial time, then every component of H is a path.

Chromatic Number

Theorem (Hoang, Kaminski, Lozin, Sawada, and Shu, 2010)

The k-colorability problem can decided in polynomial time for P_5-free graphs.

Theorem (Huang, 2013)

The k-colorability problem is NP-complete for the class of P_t-free graphs for all $k \geq 5, t \geq 6$.

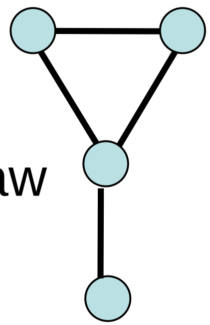
The 4-colorability problem is NP-complete for the class of P_t-free graphs for all $t \geq 7$.

Chromatic Number

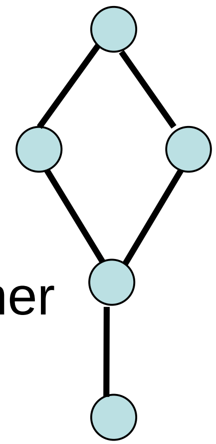
Conjecture (Huang, 2013)

The 4-colorability problem can be decided in polynomial time for P_6-free graphs.

Induced subgraphs



Paw



Banner

Chromatic Number

Theorem (Randerath, IS, and Tewes, 2002)

Every (P_6, K_3)-free graph is 4-colorable and there is a polynomial time algorithm for 4-coloring such graphs.

Theorem (Huang, 2013)

The 4-colorability problem can be solved in polynomial time for the class of (P_6, paw)-free graphs.

Chromatic Number

Theorem (Lozin and Rautenbach, 2003)

The 4-colorability problem can be solved in polynomial time for the class of $(P_6, K_{1,r})$-free graphs for any $r \geq 3$.

Theorem (Huang, 2013)

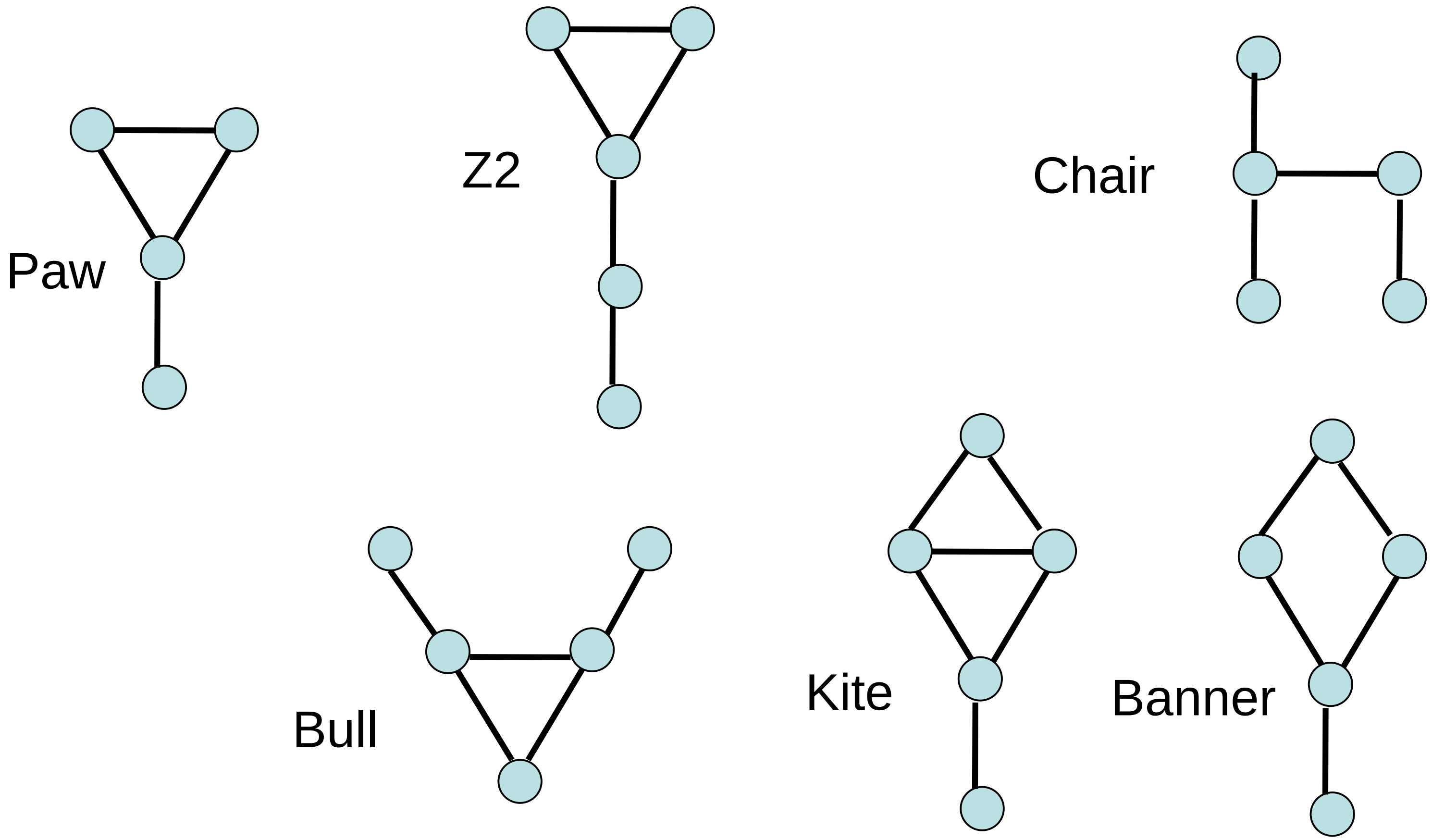
The 4-colorability problem can be solved in polynomial time for the class of (P_6, banner)-free graphs.

Chromatic Number

Theorem (Chudnovsky, Maceli, Stacho, and Zhong, 2014)

The 4-colorability problem can be solved in polynomial time for the class of (P_6, C_5)-free graphs.

Induced subgraphs



Chromatic Number

Theorem (BHKRSV, 2015)

The 4-colorability problem can be solved in polynomial time for the class of (P_6, bull, Z_2)-free graphs.

The 4-colorability problem can be solved in polynomial time for the class of $(P_6, \text{bull}, \text{kite})$-free graphs.

Theorem (BHKRSV, 2015)

The 4-colorability problem can be solved in polynomial time for the class of (P_6, chair)-free graphs.

Rainbow Colourings

najlepša **hvála**!

Thank you very much!